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Calculation of Fin Efficiency for Wet and Dry Fins

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This paper presents a quantitative evaluation of methods used to calculate fin efficiency for dry and wet fins. The exact solution for circular fins uses Bessel functions, which are tedious to evaluate. The authors provide an empirical modification to the Schmidt equation (Hong and Webb equation), which provides improved accuracy for circular fins. Existing methods to evaluate the fin efficiency of rectangular, plain plate fins are evaluated. The plate-fin methods are compared against the fin efficiency calculated by the sector method using Bessel functions. Use of the sector method with the Hong and Webb equation is recommended. Cases for which the simple "equivalent circular fin" method is a very good approximation are defined. Methods to calculate the fin efficiency of enhanced fin geometries are discussed. The authors show that the wet surface fin efficiency for any fin geometry may be calculated using a simple modification of the dry surface fin efficiency equation. Example calculations are presented, which show that the wet surface fin efficiency can be as much as 35% below the dry surface fin efficiency for high humidity conditions.

Introduction

Determination of fin efficiency is an important element in finned-tube heat exchanger analysis. To determine the air-side heat transfer coefficient, one typically obtains the *UA* value from wind tunnel test data. Such tests are normally done using water flow on the tube side under conditions for which the water-side resistance is small relative to the air-side. The tube-side heat transfer coefficient is calculated from existing empirical correlations, such as the Petukhov (1970) equation. The air-side thermal resistance is obtained by subtraction of the tube-side thermal resistance from the total thermal resistance (1/*UA*). The air-side thermal resistance, per unit air-side surface area, is the product of the air-side heat transfer coefficient and the fin efficiency. In order to extract the air-side heat transfer coefficient, the fin efficiency must be determined.

Various equations have been determined to calculate the fin efficiency. The fin efficiency calculation depends on the fin geometry, and whether there is a latent heat transfer contribution. For sensible heat transfer, ARI *Standard* 410-81 (1981) gives fin efficiency equations for different types of fin geometries under dry conditions.

Most undergraduate heat transfer texts, e.g., Incropera and DeWitt (1990) and McQuiston and Parker (1994), provide fin efficiency equations for dry, plain, and circular fins. The analytical solution for a circular fin is reported by Kern and Kraus (1972) and involves modified Bessel functions, which are tedious to evaluate. Simplified equations have been developed for circular fins. These include the empirical equation of Schmidt (1945), which will be given and discussed later. Charters and Theerakulpisut (1989) provide tenth degree polynomial curve fits of the modified Bessel functions for specific values of r_o/r_i.

This paper is also concerned with fin efficiency for dehumidification (wet fins), which involves both sensible and latent heat transfer. McQuiston (1975) developed an approximate treatment for wet surface fin efficiency for the case of a plane fin (uniform cross section). McQuiston and Parker (1994 and earlier edi-

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tions) extend the analysis to circular fins. Others have numerically solved the wet surface fin efficiency problem. Seshimo et al. (1989) and Kazwminejad et al. (1993) describe numerical solutions for circular fins. Both authors state that the fin efficiency for dehumidification is significantly less than that of a dry fin.

Based on his experiments, McQuiston (1978) reported that the heat transfer coefficient with dehumidification on plate fins is less than that for dry conditions. The paper is ambiguous on what equation was used to calculate the wet surface fin efficiency for the plate fins. However, private communication with McQuiston (1995) has indicated that he used the Schmidt (1945) equation, with *m* replaced by *M*.

This paper presents the analytical formulation of wet surface fin efficiency for circular fins. The wet surface fin efficiency is expressed in the same form as for the dry condition fin efficiency, except for the arguments in the modified Bessel functions. The argument is a function of air-humidity change over the fin, as described by McQuiston and Parker (1994).

Fin Efficiency Equation for Dry Fins

As developed by Kern and Kraus (1972), and given in ARI *Standard* 410 (1981), the fin efficiency for a circular fin (Figure 1) is given in terms of modified Bessel functions as

$$\eta_{dry} = \frac{2r_i}{m(r_o^2 - r_i^2)}\left[\frac{K_1(mr_i)I_1(mr_o) - K_1(mr_o)I_1(mr_i)}{K_1(mr_o)I_o(mr_i) + K_o(mr_i)I_1(mr_o)}\right] \tag{1}$$

Because our wet surface analysis parallels that of the dry circular fin, we have given the derivation of Equation (1) in the Appendix. Because of the calculation difficulties associated with the Bessel functions of Equation (1), Schmidt (1945) developed the following empirical equation for a circular fin

$$\eta = \frac{\tanh(mr_i\phi)}{mr_i\phi} \tag{2}$$

where

$$\phi = \left(\frac{r_o}{r_i} - 1\right)\left[1 + 0.35\ln\left(\frac{r_o}{r_i}\right)\right]$$

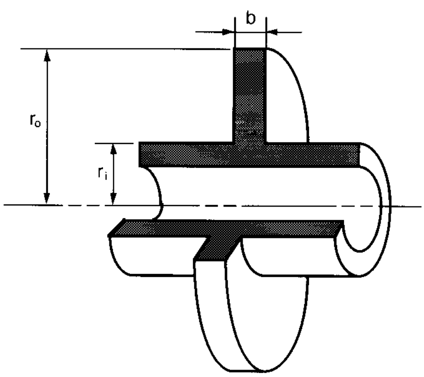

Figure 1. Circular fin with a round tube

Figure 2 compares the fin efficiency calculated by Equations (1) and (2). For $m(r_o - r_i) < 1.5$, and $r_o/r_i \le 4$, Equation (2) agrees within 2% of Equation (1). However, for $m(r_o - r_i) > 1.5$, Equation (2) overestimates η. For $3 < r_o/r_i < 5$, and $1.5 < m(r_o - r_i) < 2.5$, the errors in Equation (2) are between 5% to 18%. The error increases with increasing values of $m(r_o - r_i)$. The Schmidt equation is deemed to have unacceptable error for $r_o/r_i > 3$, and for $m(r_o - r_i) > 2.0$. It is probable that most practical fin applications would involve $r_o/r_i \le 4$ and $m(r_o - r_i) < 2.0$, for which Equation (2) would be accurate within 5%.

It is possible to propose a more accurate empirical form of the fin efficiency equation for circular fins, and one which will provide high accuracy for a much wider range of r_o/r_i and $m(r_o - r_i)$ than provided by the Schmidt equation, Equation (2). Charters and Theerakulpisut (1989) did this using polynomial curve fits of the calculated fin efficiency equation based on the solution of Equation (1). They provided 11 empirical equations for $2 \le r_o/r_i \le 4$, using 0.20 increments of r_o/r_i. This approach requires the use of 11 different polynomial equations, and interpolation for r_o/r_i values between those for which the equations exist. A more concise approximation is desirable.

The present authors have developed a variant of Equation (2) that is a more accurate approximation of Equation (1). This equation is given by

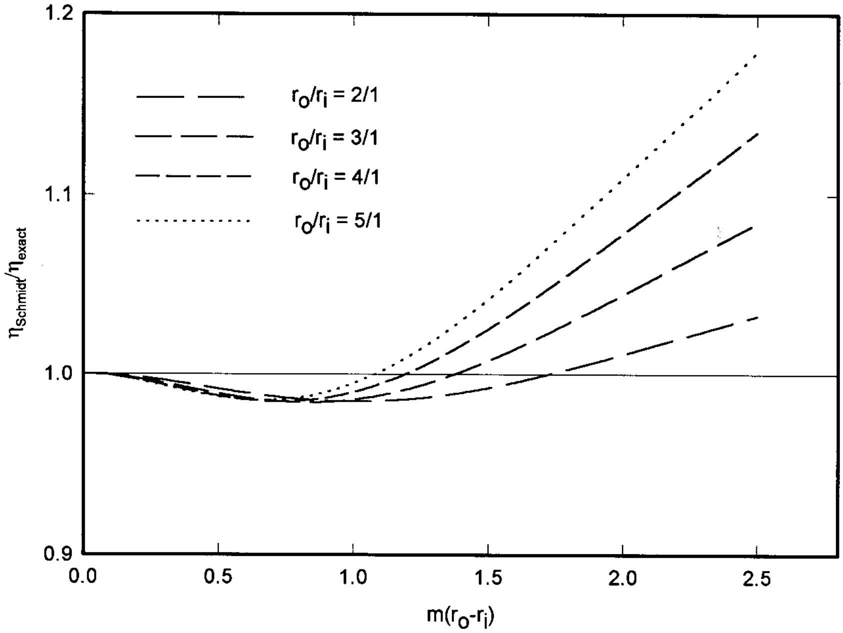

Figure 2. Fin efficiency ratio of Schmidt equation to exact solution

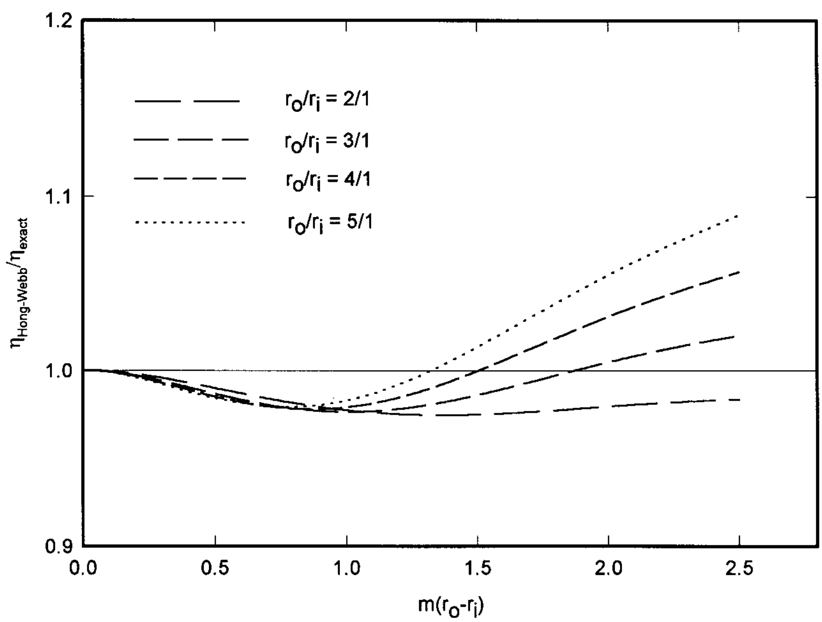

Figure 3. Fin efficiency ratio of Hong-Webb equation to exact solution

$$\eta = \frac{\tanh(mr_i\phi)\cos(0.1mr_i\phi)}{mr_i\phi} \tag{3}$$

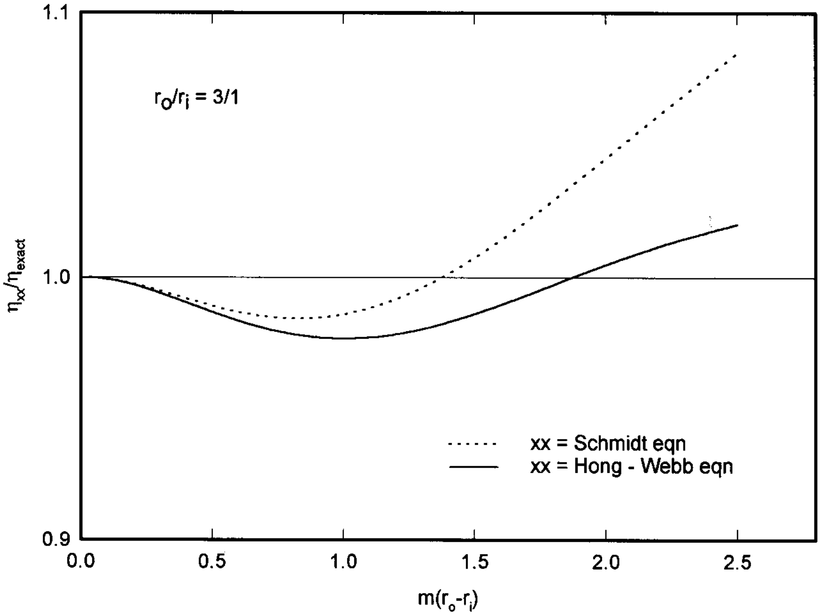

Figure 4. Fin efficiency ratio of Schmidt and Hong-Webb equation to exact solution

where the term ϕ remains the same as for Equation (2). Equation (3) is referred to as the Hong-Webb equation. Figures 2 and 3 show the ability of Equations (2) and (3) to predict the fin efficiency. Figure 3 shows the ratio of the fin efficiency predicted by Equation (3) divided by the fin efficiency predicted by Equation (1) for the range $2.0 < r_o/r_i < 5.0$. This is the range expected to be of practical interest. Equation (3) predicts the fin efficiency within 1.8% for $r_o/r_i = 3.0$, and $m(r_o - r_i) \leq 2.5$. Figure 4 compares the error of Equations (2) and (3) for $r_o/r_i = 3.0$. Figure 4 shows that the error of the Schmidt equation, Equation (2), is as much as 8% at $m(r_o - r_i) \leq 2.5$. Hence, Equation (3) shows better accuracy than the Schmidt (1945) equation over a wider range $m(r_o - r_i)$.

FIN Efficiency EQUATION for WET FINS

Under dehumidification conditions, with the fin surface temperature below the dewpoint, both sensible and latent heat transfer occurs. McQuiston (1975) and McQuiston and Parker (1994) describe how to modify the dry fin efficiency equation to account for sensible and latent effects. The 1975 analysis is limited to plane fins. However, the McQuiston and Parker (1994) presentation (p. 613) describes how to apply the concept to fins of different shapes. The McQuiston (1975) equation for plane fins is

$$\eta_{wet} = \frac{\tanh(Ml)}{Ml} \tag{4}$$

where

$$M^2 = \frac{hP}{kA_c}\left(1 + C\frac{i_{fg}}{c_{p,a}}\right)$$

The constant C shown in the above equation is the mathematical average of C_i and C_o, which are calculated as follows

$$C_i = \frac{W_{air,\,in} - W_{fin,\,base}}{T_{air,\,in} - T_{fin,\,base}}$$

$$C_o = \frac{W_{air,\,out} - W_{fin,\,base}}{T_{air,\,out} - T_{fin,\,base}}$$

$$C = \frac{C_i + C_o}{2}$$

Equation (4) is valid only if the fin cross section area is uniform. The heat conduction area increases with distance from the fin base for a circular or plate fins on round tubes. In this section, we will follow the method of McQuiston (1975) to develop a fin efficiency equation for wet, circular fins. The procedure is identical to that in the Appendix, except for replacing Equation (A-4) with Equation (5). The heat transfer rate is the sum of the sensible heat and the latent heat transfer. The latent heat is proportional to the mass transfer coefficient and the specific humidity change

$$dq_{conv} = dA_s\,[h(T - T_{air}) + K_m i_{fg}(W - W_{air})] \tag{5}$$

where $dA_s = 2dr\,(r\,\gamma)$.

Equation (5) may be simplified using the heat and mass transfer analogy given by

$$K_m = \frac{h}{c_{p,\,a}}\left(\frac{\alpha}{D}\right)^{2/3} \cong \frac{h}{c_{p,\,a}} \tag{6}$$

The value of $(\alpha/D)^{2/3}$ in Equation (6) equals 0.90 for moist air. However, it is typically approximated as 1.0 for moist air. Substitution of Equation (6) for K_m into Equation (5) allows the mass transfer coefficient K_m to be written in terms of the sensible heat transfer coefficient h.

Following McQuiston (1975, 1994), the specific humidity W is assumed to be a linear function of the temperature and may be written as

$$W - W_{air} = C(T - T_{air}) \tag{7}$$

The constant C can be determined using the psychometric chart for specific operating conditions. This C-value also appears in the definition for M in Equation (4). Substituting Equations (6) and (7) into the second term of Equation (5) yields

$$dq = dA_s h\,(T - T_{air})\left(1 + C\frac{i_{fg}}{c_{p,\,a}}\right) \tag{8}$$

Replacing the convection term in Equation (A-4) with Equation (8) yields

$$\begin{aligned} \frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} &= M^2\theta \\ M^2 = \frac{2h}{kb}\left(1 + \frac{i_{fg}}{c_{p,\,a}}C\right) &= m^2(1 + \beta) \end{aligned} \tag{9}$$

where $\theta = T - T_{air}$ and $\beta = i_{fg}C/c_{p,a}$

The boundary conditions appropriate for Equation (9) are: (1) at $r = r_i$, $\theta_b = T_b - T_{air}$, and (2) at $r = r_o$, $d\theta/dr = 0$. The fin temperature distribution for the wet condition is obtained by solving Equation (9) with

the given boundary conditions. Equation (9) is identical to Equation (A-5) except for the arguments of the modified Bessel function.

$$\theta_{wet}(r) = \theta_b\left[\frac{K_1(Mr_o)I_0(Mr) + I_1(Mr_o)K_0(Mr)}{K_1(Mr_o)I_0(Mr_i) + K_0(Mr_i)I_1(Mr_o)}\right] \tag{10}$$

The wet surface fin efficiency is defined as the ratio of an actual heat transfer rate to the maximum possible heat transfer rate. The heat transfer rate with dehumidification is the summation of the sensible and latent terms as shown by Equation (5). Hence, the wet surface fin efficiency is calculated as

$$\eta_{wet} = \frac{q_{actual}}{q_{max}} = \frac{\int_{r_i}^{r_o}[h(T_f - T_{air}) + K_m i_{fg}(W_f - W_{air})]dr}{\int_{r_i}^{r_o}[h(T_b - T_{air}) + K_m i_{fg}(W_b - W_{air})]dr} \tag{11}$$

The integrands in Equation (11) may be simplified using Equation (8). The terms, except for the temperature difference, are canceled out in Equation (11). Hence, the numerator is a function only of θ_f, and the denominator has only θ_b term, as shown in Equation (A-8). Therefore, calculation of θ_b and θ_f is needed to obtain the wet surface fin efficiency.

$$\theta_b = \left[\frac{K_1(Mr_o)I_0(Mr_i) + K_0(Mr_i)I_1(Mr_o)}{I_1(Mr_o)}\right]C_2 \tag{12}$$

$$\theta_f = C_2\frac{2r_i}{M(r_o^2 - r_i^2)}\left[\frac{K_1(Mr_i)I_1(Mr_o) - K_1(Mr_o)I_1(Mr_i)}{I_1(Mr_o)}\right] \tag{13}$$

Dividing Equation (12) by Equation (13) yields the wet surface fin efficiency given by

$$\eta_{wet} = \frac{2r_i}{M(r_o^2 - r_i^2)}\left[\frac{K_1(Mr_i)I_1(Mr_o) - K_1(Mr_o)I_1(Mr_i)}{K_1(Mr_o)I_0(Mr_i) + K_0(Mr_i)I_1(Mr_o)}\right] \tag{14}$$

Further, Equation (14) may be calculated by Equation (1), the Schmidt (1945) equation, Equation (2), or by the Hong-Webb equation, Equation (3), if the parameter m is replaced by M. The only difference is that Equation (14) contains $M = m(1 + \beta)^{0.5}$, as opposed to the parameter m in Equations (1), (2) or (3). Assuming that the h-value is the same for a given dry and wet surface, the value of M^2 for a wet surface will be greater than m^2 for a dry surface.

Results and Discussion

Figure 5 shows the calculated fin efficiency for circular fins under dry and wet conditions. The dry fin efficiency is calculated using Equation (1) and by Equation (14) for the wet surface. The calculations were performed for the specific conditions shown on Figure 5. Because a wet surface will operate at a higher value of $m(r_o - r_i)$ than a dry surface, one should not compare the wet and dry surface fin efficiencies at the same value of $m(r_o - r_i)$. If the dry surface operates at a given $m(r_o - r_i)$, the wet surface will operate at an $m(r_o - r_i)$ that is $(1 + \beta)^{0.5}$ times larger than that of the dry surface.

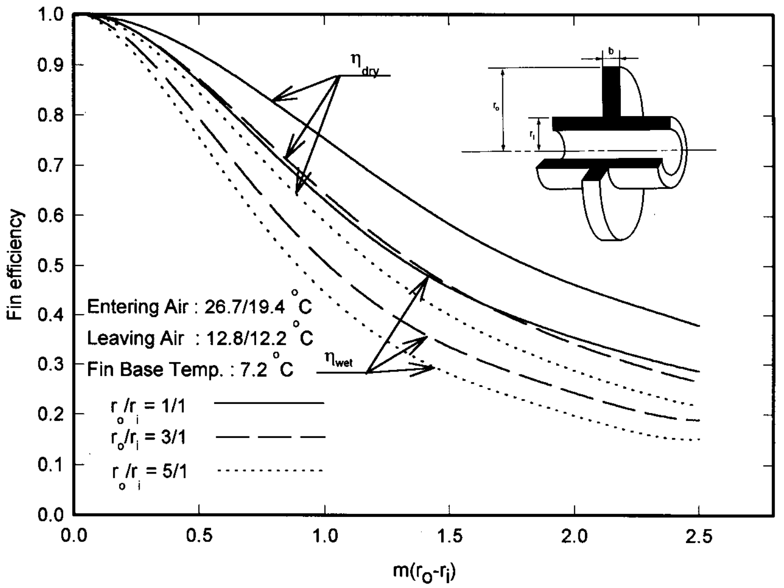

Figure 5. Fin efficiency under wet condition

Example Calculation

To illustrate the difference between wet and dry surface fin efficiency, for specified operating conditions, we have prepared an example calculation. The tube pitch and tube diameter used in this example calculation are 39.73 mm (1.564 inch) and 15.88 mm (0.625 inch), respectively, giving $r_o/r_i = 2.5$. The fin has $t = 0.127$ mm (0.005 inch), and $k = 190.37$ W/(m·K) (110 Btu/h·ft·°F). The fin base temperature is kept constant at 7.2°C, and the dry surface heat transfer coefficient is $h = 170.33$ W/(m^2·K) (30 Btu/h·ft^2·F). The dry and wet bulb temperatures of the entering air are 26.7°C and 23.9°C, respectively. The dry and wet bulb temperatures of the leaving air are 12.8°C and 12.2°C, respectively. Figure 6 shows the fin efficiency curve calculated using Equation (1) for $r_o/r_i = 2.5$. For the dry surface, $m(r_o - r_i) = 1.4$. Thus, one finds $\eta_{dry} = 0.52$ by reading the fin efficiency curve on Figure 6. The next step is to calculate the wet surface fin efficiency. The β-term depends on the condensing load. For the stated operating conditions, we find that $C = 4.99 \times 10^{-4}$. Thus, $\beta = i_{fg}C/c_{p,a} = 1.23$ and the abscissa value of Figure 6 is $m(r_o - r_i)(1 + \beta)^{0.5} = 2.1$. Figure 6 shows that the wet surface fin efficiency is 0.34. Therefore, the wet surface fin efficiency (0.34) is 35% smaller than the dry surface value (0.52).

If one uses the Schmidt equation to calculate the wet surface fin efficiency, one replaces m with M, which results in $m(r_o - r_i)(1 + \beta)^{0.5} = 2.1$. This is slightly beyond the maximum recommended range $[m(r_o - r_i) \leq 2.0]$ for use of the Schmidt equation.

Effect of T_{wb} on η_{wet}

Figure 7 shows the effect of wet-bulb temperature T_{wb} on η_{wet}. Increasing T_{wb} of the entering air increases the condensation load on the fins. The wet surface fin efficiency is as much as 35% below the dry surface value (at the same abscissa value) when the dry and wet-bulb temperatures of the entering air are 26.7°C and 23.9°C, respectively. The dry and wet-bulb temperatures of the leaving air are 12.8°C and 12.2°C, respectively. The influence of the entering air humidity shown in Figure 7 is in good agreement with the results of Kazeminejad et al. (1993).

Figure 8 shows the ratio of the wet surface fin efficiency for plane fins, Equation (4), to that for circular fins, Equation (14), for $\beta = 0.81$. Figure 8 shows that the wet surface fin efficiency of circular fins is less than that of plane fins for the same $M(r_o - r_i)$ values.

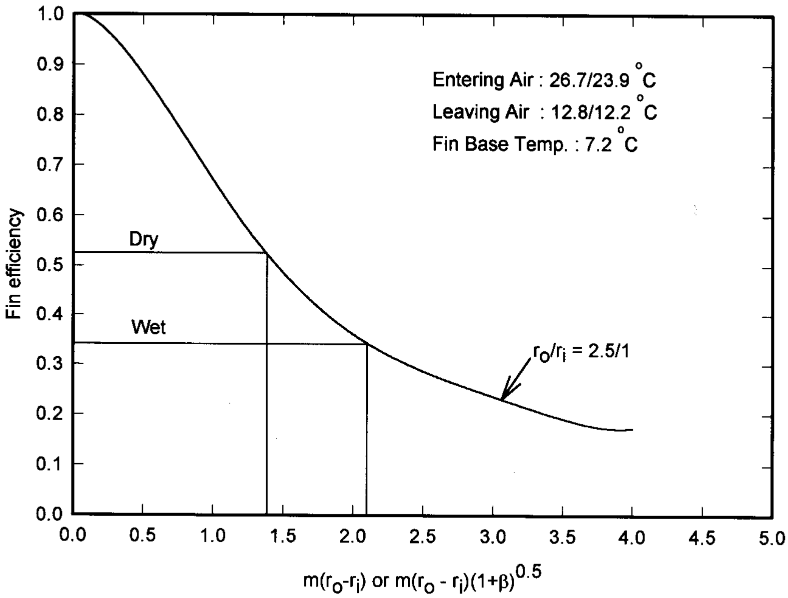

Figure 6. Fin efficiency for dry and wet condition

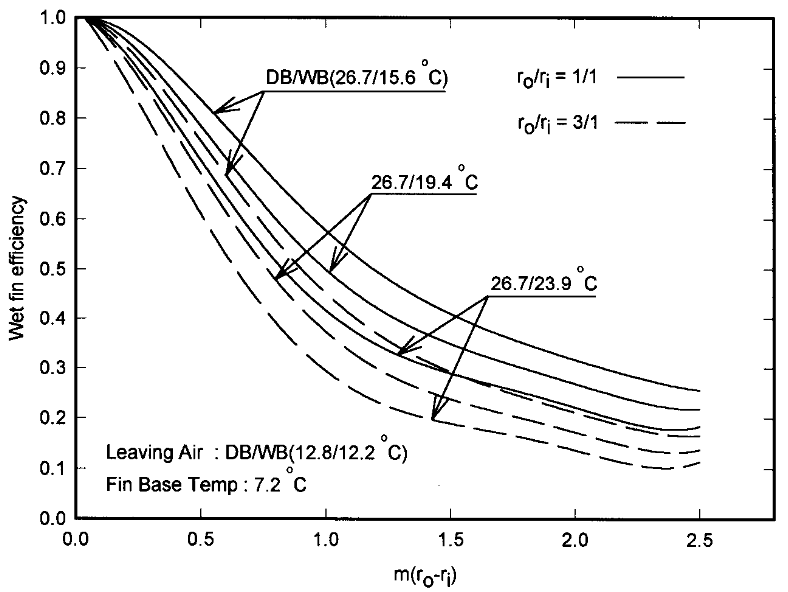

Figure 7. Entering air humidity effect on fin efficiency

Other Fin Shapes

The present analysis may be extended to other fin (and tube) shapes. Such shapes of interest for round tubes are shown in Figure 1. These include the spine fin (Figure 9a), the continuous-plate fin (Figure 9b), and “slit” plate fins (Figure 9c). Figure 9d shows offset-strip fins on “flat” tubes, which are typically used in automotive or brazed aluminum heat exchangers.

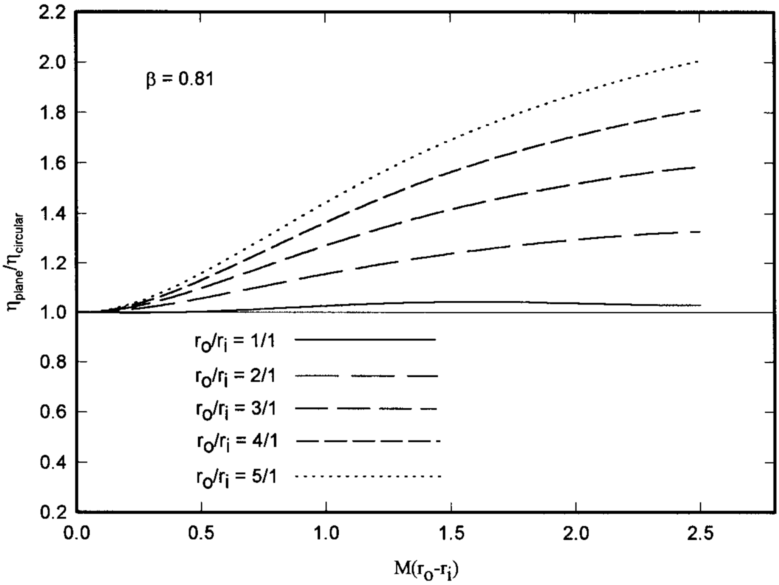

Figure 8. Wet surface fin efficiency ratio, plane-to-circular fins

Plain Plate-Fins

The simplest method to calculate the fin efficiency of the Figure 9b plan plate fins is the "equivalent circular fin." Using this method, one calculates the fin efficiency of a circular fin having the same fin area as the plate fin. Zabronsky (1955) has shown that this approach is quite good for square fins. However, no formal investigation of the errors associated with this method have been reported for fins of a rectangular shape. Rich (1966) shows that the sector method is the best method to predict the fin efficiency of the rectangular shaped plain plate fins. Rich evaluates the fin efficiency for each sector using a power-series form of the solution to the differential equation [Equation (A-5)]. Because this method requires a computer program, it is not appropriate for hand calculation. As discussed by McQuiston and Parker (1994, p. 613), Schmidt (1945) proposed that his circular fin equation [Equation (2)] method may be applied to the Figure 9b plain-plate fin. Schmidt defined the radius r_o of an equivalent circular fin that will have the same fin efficiency as that of the plate fin. This equivalent r_o is defined by

$$\frac{r_o}{r_i} = 1.28\psi\ (\beta - 0.2)^{1/2} \tag{15}$$

where the terms ψ and β are defined as

$$\psi = W/r_i \qquad \beta = L/W \tag{16}$$

with the L and W defined on Figure 9b.

Because of the errors associated with the Schmidt method for circular fins, as shown on Figure 2 and Figure 4, the authors have evaluated the accuracy of Equation (15), the sector method using the Hong-Webb equation [Equation (3)], and the "equivalent circular fin" method for square and rectangular plate fins. The accuracy of these equations are measured relative to the fin efficiency calculated by the sector method, as suggested by Rich (1966), using Equation (1). The results are shown on Figures 10, 11, and 12. These figures show the ratio of the calculated fin efficiency to that calculated by the sector method with

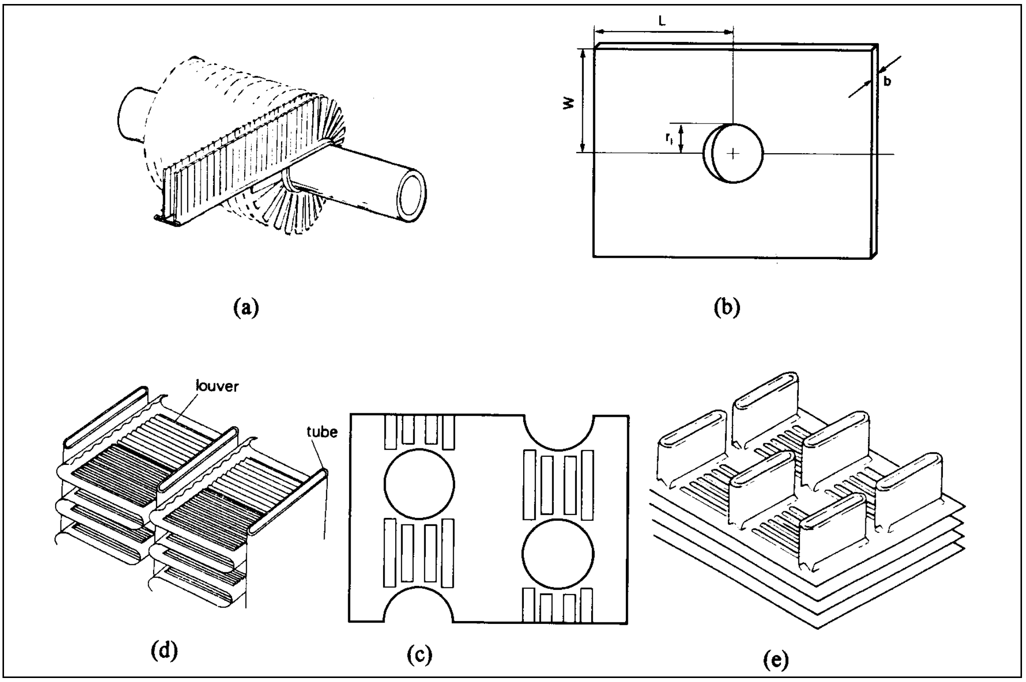

Figure 9. (a) Spine Fin, (b) Continuous-Plate Fin, (c) Slit Fin, (d) Corrugated louver fins on flat tube, (e) Louver plate fins on flat tube

Equation (1). Calculations were performed for W/r_i = 1.0, 3.0, and L/W = 1.0, 3.0, and 0.866 (equilateral triangular pitch tube layout). These geometry variants were selected to span the range of practical interest. In general, the Hong-Webb method [Equation (3)] shows very good agreement with the sector method. Except for the rectangular fin with W/r_i = 3.0, and L/W = 3 (Figure 11), the agreement is within 3.0%. The Schmidt equation shows errors as high 10% (or more) for most cases. The "equivalent circular fin" method is generally competitive with the Schmidt method; both of these methods show unacceptable error for the L/W = 3.0 rectangular fins (Figure 11).

Enhanced Fin Geometries

The Figure 9a spine fin may be analyzed as a plane fin using Equation (4) with M replaced by m. None of the previously discussed methods may be applied to the Figure 9c slit-plate fin. Because the slits cut the heat conduction path from the tube, a radial heat flow pattern will not exist. As more louvers are applied to the fin surface, the conduction path is further impaired, and the fin efficiency will decrease. The only practical way to accurately calculate the fin efficiency of the Figure 9c fin is to numerically solve the heat conduction equation for the fin geometry of interest. Clearly, use of the circular fin efficiency equation will over predict the fin efficiency. The Figure 9d corrugated, louvered fins on flat tubes may be predicted using the fin efficiency equation for plane fins [Equation (4) with M replaced by m]. The fin efficiency for the Figure 9e louvered plate-fins on flat tubes cannot strictly be calculated using the equation for plane fins. However, if the spacing between tubes in the air flow direction is small, the plane fin equation may be an acceptable approximation. The actual fin efficiency will be less than that given by the plane fin equation.

Typical air-conditioning applications would be approximately spanned by $1.0 \le L/W \le 1.5$ and (rectangular fins) and L/W = 0.866 (equilateral triangular tube layout) with W/r_i = 3. For these cases, Figures 10 and 12 show that the simple "equivalent circular fin" is a very good approximation, with errors less than 3%.

For the wet surface condition, the fin efficiency may be calculated using the appropriate dry surface fin efficiency equation for the specific geometry with the parameter m replaced by M.

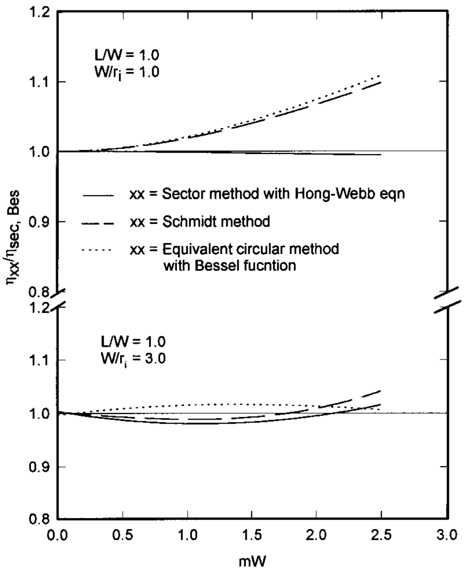

Figure 10. Fin efficiency ratio of Hong-Webb, equivalent circular, and Schmidt method to sector method with Bessel function (for *L*/*W* = 1.0)

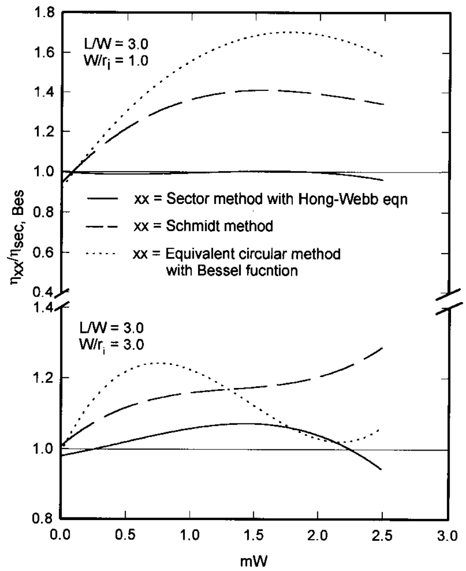

Figure 11. Fin efficiency ratio of Hong-Webb, equivalent circular, and Schmidt method to sector method with Bessel function. (For *L*/*W* = 3.0)

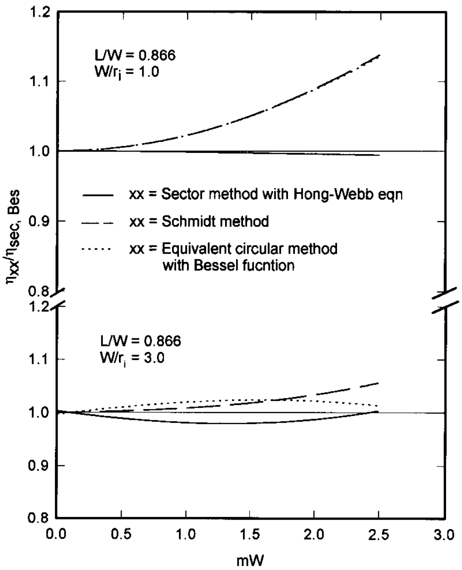

Figure 12. Fin efficiency ratio of Hong-Webb, equivalent circular, and Schmidt method to sector method with Bessel function. (For *L*/*W* = 0.866)

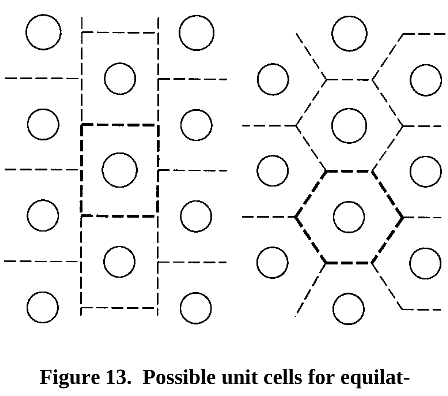

Figure 13. Possible unit cells for equilateral triangular tube layout: (a) Rectangular cells (b) Hexagonal cells

Selection of Unit Cell for Sector Method

Figure 13 shows possible methods to apply the sector method for an equilateral staggered tube layout. The left side of the figure shows rectangular unit cells and the right side shows hexagonal unit cells. Heat flux symmetry suggests that use of the hexagonal unit cell is best for staggered tubes on an equilateral pitch. For other staggered tube layouts, a symmetric six-sided cell may also be defined and is recommended for fin efficiency calculation. The authors have found that the calculated fin efficiency of the hexagonal (or six-sided) cell is greater than that of the alternate rectangular cell. For inline tube layouts, the only possible choice is rectangular unit cells.

Conclusions

1. This paper presents a quantitative evaluation of methods used to calculate fin efficiency for dry (and wet) fins.
2. For typical air-conditioning applications [$1.0 \le L/W \le 1.5$ and (rectangular fins) and $L/W = 0.866$ (equilateral triangular tube layout) with $W/r_i = 3$], the simple "equivalent circular fin" is a very good approximation, with errors less than 3%.
3. The Schmidt (1945) method shows errors as large as 18% for circular fins, and relatively large errors for rectangular fins and square fins having $W/r_i \cong 1$.
4. We have developed an empirical modification to the Schmidt equation [Equation (2)], which is accurate within 9% for circular fins.
5. For rectangular fins and square fins with $W/r_i \cong 1$, use of the sector method with Equation (3) is recommended.
6. For the same air velocity, the wet surface fin efficiency may be calculated using the same equation as for a dry surface, if the parameter m is replaced by M. Thus, the fin efficiency is evaluated at an argument that is $(1 + \beta)^{0.5}$ larger than that of the dry surface. This acts to reduce the wet surface fin efficiency, as compared to that of the dry surface. Further, the difference between wet and dry surface fin efficiency increases as the latent load increases.
7. Example calculations are presented, which show that under high humidity conditions, the wet surface fin efficiency can be as much as 35% below the dry surface fin efficiency.

Nomenclature

A area, m^2

b	fin thickness, m
c_p	specific heat at constant pressure, J/(kg·K)
C	average slope of dW/dT from psychometric chart, C_i, (at entering air temperature), C_o (at leaving air temperature), K^{-1}
D	molecular diffusion coefficient, m^2/s
h	convection (sensible) heat transfer coefficient, $W/(m^2 \cdot K)$
i_{fg}	latent heat of vaporization, J/kg
I_o	modified Bessel function of the first kind, order 0
I_1	modified Bessel function of the first kind, order 1
K_o	modified Bessel function of the second kind, order 0
K_1	modified Bessel function of the second kind, order 1
K_m	mass transfer coefficient, m^2/s
k	thermal conductivity of fin, W/(m·K)
m	parameter defined as $m = (2h/kb)^{0.5}$, dimensionless
M	parameter defined as $M = m\,(1+\beta)^{0.5}$, dimensionless
P	circumference, m or ft
q	heat transfer rate, W
r_o	distance from the center of the tube to the fin tip, m
r_i	distance from the center of the tube to the fin base, m
T	temperature, °C
W	specific humidity, kg steam/kg dry air, dimensionless
W	width of rectangular fin, m
α	thermal diffusivity, m^2/s
η	fin efficiency, dimensionless
γ	angle in a control volume shown in Figure 14, degrees
θ	temperature difference, K
β	parameter defined as $\beta = Ci_{fg}/c_{p,a}$, dimensionless

Subscripts

a	air
avg	average
b	base
Bes	Bessel
c	cross sectional
conv	convection
dry	dry condition
e	empirical
f	fin
wet	wet condition
exact	fin efficiency based on the modified Bessel function

References

ARI. 1981. Standard for Forced Circulation Air Cooling and Air-Heating Coils. *ARI Standard 410-81*. Air-Conditioning and Refrigeration Institute, Arlington, VA.

Charters, W.W.S., and S. Theerakulpisut. 1989. Efficiency Equations for Constant Thickness Annular Fins. *Int. Comm. Heat Mass Transfer* 16: 547-558.

Kazeminejad, H., M.A. Yaghoubi, and M. Sepehri. 1993. Effect of Dehumidification of Air on the Performance of Eccentric Circular Fins. *Proc. Institution Mech. Engineers*, 207: 141-146.

Kern, D.Q., and D.A. Kraus. 1972. *Extended Surface Heat Transfer*. New York: McGraw-Hill.

McQuiston, F.C. 1975. Fin Efficiency with Combined Heat and Mass Transfer. *ASHRAE Transactions* 81(1): 350-355.

McQuiston, F.C. 1978. Heat, Mass, and Momentum Transfer Data for Five Plate-Fin-Tube Heat Exchangers. *ASHRAE Transactions* 84(1): 266-293.

McQuiston, F.C., and J.D. Parker. 1994. *Heating, Ventilating, and Air Conditioning*, Fourth Ed. New York: John Wiley & Sons.

McQuiston, F.C. 1995. Private facsimile communication with R.L. Webb, October 26.

Rich, D.G. 1966. The Efficiency and Thermal Resistance of Annular and Rectangular Fins. *Proc. of International Heat Transfer Conference* 3: 281-289.

Senshu, T., T. Hatada., and K. Ishibane.1981. Heat and Mass Transfer Performance of Air Coolers Under Wet Conditions. *ASHRAE Transactions* 87(2): 109-115.

Seshimo, Y., K. Ogawa , K. Marumoto, and M. Fujii. 1989. Heat and Mass Transfer Performances on Plate Fin and Tube Heat Exchangers with Dehumidification. *Heat Transfer-Japanese Research* V-18 (5): 79-94.

Schmidt T.E. 1945-46. La Production Calorifique des Surfaces Munies D'ailettes. *Bulletin De L'Institut International Du Froid* Annexe G-5.
Webb R.L.1994. *Principles of Enhanced Heat Transfer*, pp. 520-533. New York: John Wiley & Sons.
Zabronsky, H. 1955. Temperature Distribution and Efficiency of a Heat Exchanger using Square Fins on Round Tubes. *ASME Transactions, Journal of Applied Mechanics*, 77: 119-122.

Appendix

Dry Surface Fin Efficiency Equation

A steady state energy balance on the Figure 14 control volume gives

$$dq_r - (dq_{r+dr} + dq_{conv}) = 0 \quad \text{(A-1)}$$

For a constant fin thickness, conduction heat transfer changes with distance from the fin base as given by

$$dq_{r+dr} = dq_r + \frac{d}{dr}(dq_r)dr, \quad dq_r = -k\,A_c(r)\frac{dT}{dr} \quad \text{(A-2)}$$

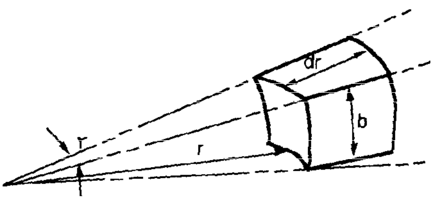

Figure 14. Control volume in a circular fin

Convection heat transfer also changes with distance from the fin base as given by

$$dq_{conv} = -\frac{d}{dr}(dq_r)dr = -\left[\frac{d}{dr}\left[-k(r\gamma b)\frac{dT}{dr}\right]\right]dr \quad \text{(A-3)}$$

$$dq_{conv} = 2A_s\,h\,(T - T_{air}), \quad A_s = dr(r\gamma) \quad \text{(A-4)}$$

Combining Equation (A-1) through (A-4) will give the differential equation

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - m^2\theta = 0, \quad \text{where } m^2 = \frac{2h}{kb}, \quad \theta = T - T_{air} \quad \text{(A-5)}$$

The boundary conditions are: (1) at $r = r_i$, $\theta_b = T_b - T_{air}$, (2) at $r = r_o$, $d\theta/dr = 0$. The solution to Equation (A-5) is shown in Equation (A-6)

$$\theta(r) = \theta_b\left[\frac{K_1(mr_o)I_0(mr) + I_1(mr_o)K_0(mr)}{K_1(mr_o)I_0(mr_i) + K_0(mr_i)I_1(mr_o)}\right] \quad \text{(A-6)}$$

Fin efficiency is defined as the ratio of actual heat transfer rate to the possible maximum heat transfer rate. It is normally assumed that heat transfer coefficient is constant over the surface.

$$\eta_{dry} = \frac{q_{actual}}{q_{max}} = \frac{\int_{r_i}^{r_o} h(T_f - T_{air})dr}{\int_{r_i}^{r_o} h(T_b - T_{air})dr} \quad \text{(A-7)}$$

Because the fin temperature is a function of the distance from the base, it is not easy to solve the above equation analytically. Assuming that the temperature of the fin can be replaced with the average fin temperature calculated from the fin temperature profile over the fin area, the fin efficiency equation is simplified as in Equation (A-8).

$$\eta_{dry} = \frac{T_{f,avg} - T_{air}}{T_b - T_{air}} = \frac{\theta_f}{\theta_b} \tag{A-8}$$

From Equation (A-8) and the solution of Equation (A-6), the fin efficiency under dry condition is expressed by

$$\eta_{dry} = \frac{2r_i}{m(r_o^2 - r_i^2)}\left[\frac{K_1(mr_i)I_1(mr_o) - K_1(mr_o)I_1(mr_i)}{K_1(mr_o)I_0(mr_i) + K_0(mr_i)I_1(mr_o)}\right] \tag{A-9}$$

Equation (A-9) is also shown in ARI *Standard* 410 (1981) with different parameters.